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Short Papers

Measurement of Dielectric Materials Using a Cutoff Circular-Waveguide Cavity

J. HANFLING AND L. BOTTE

Abstract—**A technique is presented for accurately determining the dielectric constant of microwave materials. The concept is to resonate a cutoff circular-waveguide cavity by inserting the dielectric-disk sample. Unlike most dielectric measurement techniques which rely on perturbation methods, this one determines the dielectric constant from the absolute measurement of the resonant frequency. Also, the use of a cutoff cavity prevents false dielectric constant readings by eliminating spurious resonances.**

I. Introduction

A cutoff circular-waveguide cavity is used for accurate and convenient measurement of the dielectric constant of microwave materials. The technique is applicable to all materials which can be formed into a circular disk.

The concept consists of locating a dielectric disk transversely at the center of a short-circuited circular-waveguide cavity. The dominant resonance of the cavity is for the TE_{11} mode even though the unfilled portion of the cavity is below cutoff. The dielectric constant of the disk material is determined from the resonant frequency. In practice the cavity is not short-circuited, but is weakly coupled to rectangular waveguide by coupling holes, as shown in Fig. 1(a). The features of the technique are the following: there are no higher mode resonances, the end effects introduced by coupling into and out of the cavity are accountable, and the samples are large providing good accuracy and reliability in determining the dielectric constant.

Using the parameters in Fig. 1(b), the formulas for determining the dielectric constant will be derived. Then the measured results and accuracies will be described.

II. Derivation of Formulas

In order to determine the dielectric constant of a disk, a relation between the measured cavity resonant frequency f_0 and the dielectric constant K of the disk is established. The desired formula is obtained by means of the "transverse-resonance" procedure [1], [2]. This procedure is valid since the generator and load impedances are loosely coupled to the disk; therefore, only reactive portions of the cavity need be considered. Referring to Fig. 1(b), the transverse-resonance condition is

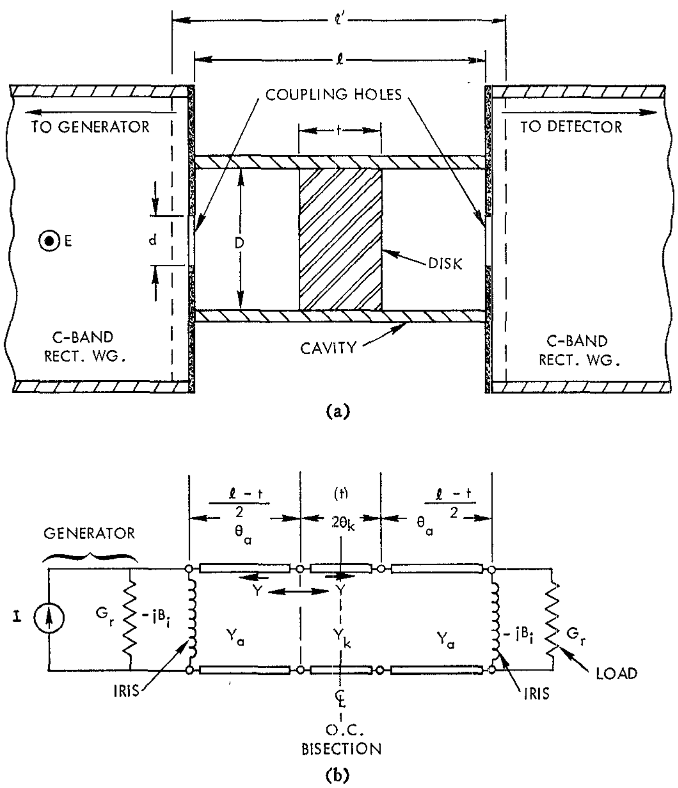

Fig. 1. (a) Cavity configuration. (b) Equivalent circuit of cavity.

$$\overleftarrow{Y} + \overrightarrow{Y} = 0. \quad (1)$$

When the center of the cavity is an open-circuit (OC) bisection, then the right-hand term in (1) becomes

$$\overrightarrow{Y} = jY_k \tan\theta_k \quad (2)$$

and the left-hand term in (1) is

$$\overleftarrow{Y} = -jY_a \cot\theta_a' \quad (3)$$

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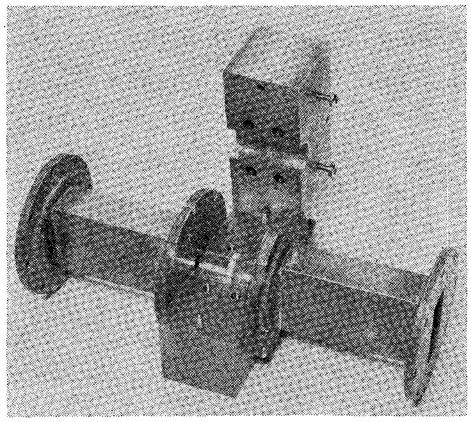

Fig. 2. Dielectric test cavity with disk inserted.

where θ_a' is determined by the effective location of the short-circuit plane and, therefore, includes the effect of the coupling holes.

The particular case of interest here is when the air-filled portion of the cavity is below cutoff at the resonant frequency of the entire loaded cavity. In this case the higher order circular-waveguide modes which are excited will decay faster than the dominant TE_{11} mode; therefore, only the dominant mode will be of significance.

Substituting (2) and (3) into (1) gives

$$jY_k \tan \theta_k = +jY_a \cot \theta_a' \tag{4}$$

where the disk wave admittance and electrical length are

$$Y_k = \frac{\lambda}{\lambda_{gk}} Y_0 \qquad \theta_k = \frac{\pi}{\lambda_{gk}} t \tag{5}$$

and the cutoff-guide wave admittance and electrical length are

$$Y_a = -j \frac{\lambda}{|\lambda_{ga}|} Y_0 \qquad \theta_a' = -j \frac{\pi}{|\lambda_{ga}|} (l' - t). \tag{6}$$

Therefore,

$$\frac{\cot \theta_k}{\theta_k} = \frac{|\lambda_{ga}|}{\pi t} \tanh \theta_a'. \tag{7}$$

Considering the case where the thickness of the disk is between zero and a quarter wavelength there will be a solution for (7). Using the measured resonant frequency of the cavity, the right-hand side of (7) can be numerically evaluated. Then by a trial-and-error procedure θ_k can be determined. The dielectric constant can then be determined from (8):

$$K = \lambda^2 \left[\left(\frac{1}{\lambda_c}\right)^2 + \left(\frac{\theta_k}{\pi t}\right)^2 \right]. \tag{8}$$

III. Measured Results

For an array-element application, a fixture has been developed to test disks in the range of 0.900-in diameter. The fixture shown in Fig. 2 consists of a circular-waveguide cavity of 0.900-in diameter and 1.895-in length, which is coupled at each end to C-band rectangular waveguides by coupling holes of 0.395-in diameter. For the test the cavity is opened and the ceramic disk is inserted at the center. The cavity is then closed and bolted. Contact between the cavity and the rectangular waveguide is provided by bolting. It is important to note that the two halves of the cavity are oriented so that no current crosses the parting line. Also, the cavity parting surfaces have been milled down by 0.010 to prevent air gaps from occurring between the top and bottom of the disks and the cavity walls.

With the cavity connected to a matched generator and detector (isolators should be used), the resonant frequency f_0 is measured by observing the output response. The Q of this brass cavity with a ceramic disk was approximately 3000, giving a sharply defined output response. The response demonstrates the absence of higher mode resonances over at least a 20-percent band.

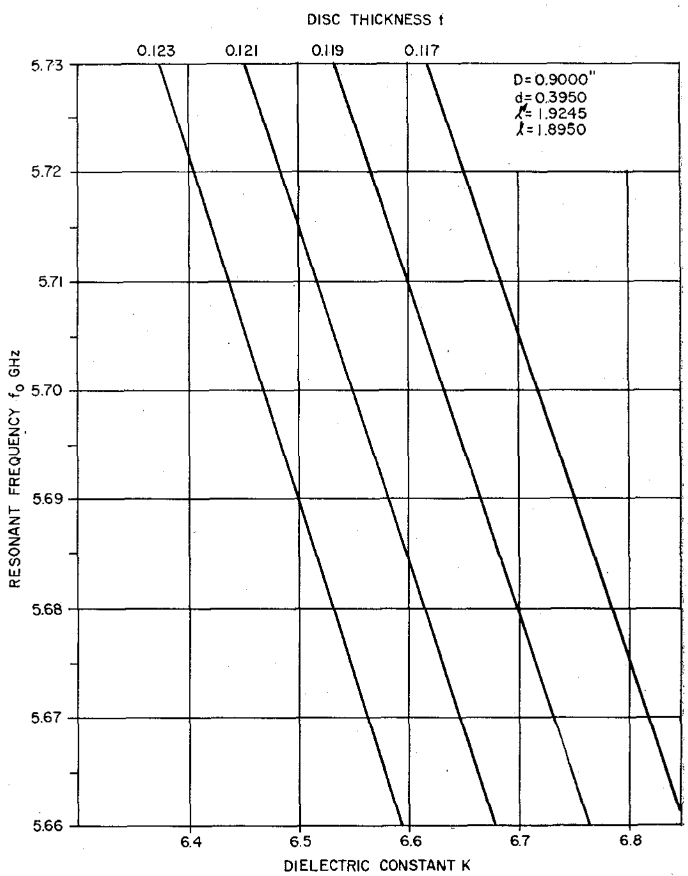

Fig. 3. Cavity calibration for 0.900-in diameter beryllia disks.

TABLE I

Sensitivities and Accuracies for Dielectric-Constant Measurements of 0.117-in Beryllia Disks

Parameter	Sensitivity (ΔK/unit)	Parameter Accuracy	Resultant Accuracy ΔK
Frequency f_0	0.0035/MHz	0.02 percent	0.0035
Cavity length l	0.0002/mil	0.002 in (end effects)	0.0004
Cavity and disk diameter D	0.010/mil	0.01 percent	0.001
Disk thickness t	0.045/mil	0.1 percent	0.0045
Air gaps ΔD_d	0.013/mil	0.0002 in (maximum gap)	0.0026
			0.0063 (rss)

Fig. 3 shows curves relating dielectric constant to resonant frequency with the disk thickness as a parameter. The curves were obtained from (7) and (8) for beryllia disks. Over the regions plotted the curves approximate straight lines.

The determination of the dielectric constant is affected by measurement errors. The effects of the significant errors for 0.117-in thick beryllia disks are given in Table I. Presented are the sensitivities to errors in the measured parameters and the resultant accuracies achievable with the typical test equipment and techniques. The cavity resonant frequency was measured at the point of maximum output using a frequency counter; values for the other parameters were determined physically. The root sum square (rss) of the 3σ errors in the measurement of frequency, cavity length, disk diameter, disk thickness, and in calibrating out the effects of air gaps results in an overall error in the computation of the dielectric constant of 0.006 or approximately a 0.1-percent accuracy.

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A Study of Microwave Leakage Through Perforated Flat Plates

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***Abstract*—A simple formula useful for predicting leakage through a circular hole array in a metallic flat plate is presented. A correction is given for plate thickness. The formula is applicable to arrays having either a 60° (staggered) or 90° (square) hole pattern, but is restricted to the case of 1) an obliquely incident plane wave with the E field polarized normal to the plane of incidence, and 2) large transmission loss. When theoretical values were compared to experimental data obtained on test samples having transmission losses greater than 20 dB, the agreement between theory and experiment was typically better than 1 dB at S band and 2 dB at X band.**

Introduction

To those involved with the development of low-noise antennas for deep-space communications and radio astronomy, the subject of leakage through antenna mesh materials is of great interest. This subject is also of interest to those concerned with microwave radiation hazards due to leakage through various types of mesh materials. Meshes have many applications; some examples are reflective surfaces on antennas, Fabry-Perot interferometers, microwave oven doors, RF screen rooms, and RF protective garments.

Meshes are usually of two types: 1) meshes formed by wire grids and 2) meshes formed by round holes in a flat metallic plate. A significant amount of experimental and theoretical work has been done on microwave reflectivity and transmission properties of wire grid type meshes [1]–[6]. Kaplun *et al.* [2] and Mumford [4] present curves useful for predicting transmission through wire grid meshes at normal incidence. To this author's knowledge, similar types of curves are not available for predicting transmission through flat plate meshes having round hole perforations. Theoretical work has been done by Chen [7] on the general problem of reflection and transmission properties of a thin conducting sheet perforated periodically with circular holes. His treatment applies to a general case of arbitrary incidence and polarization, but is mainly intended to be useful for investigating behavior in the resonance region where transmission losses are small.

It is the purpose of this correspondence to present a simple theoretical formula that can be useful for predicting transmission through circular hole arrays when the transmission losses are high (10 dB or greater). A correction is included for plate thickness. This formula is verified by experimental data obtained by both waveguide and free-space measurement techniques.

Theoretical Formula

For the case of a normally incident plane wave, an array of small holes in a thin metallic sheet behaves as an inductive susceptance in shunt with a TEM mode transmission line. Assuming the array has no resistive losses, the normalized shunt admittance is [8], [9]

$$\frac{Y}{Y_0} = -j\left(\frac{3ab\lambda_0}{\pi d^3}\right) \tag{1}$$

where

- a, b spacings between holes (Fig. 1)
- d hole diameter
- λ_0 free-space wavelength

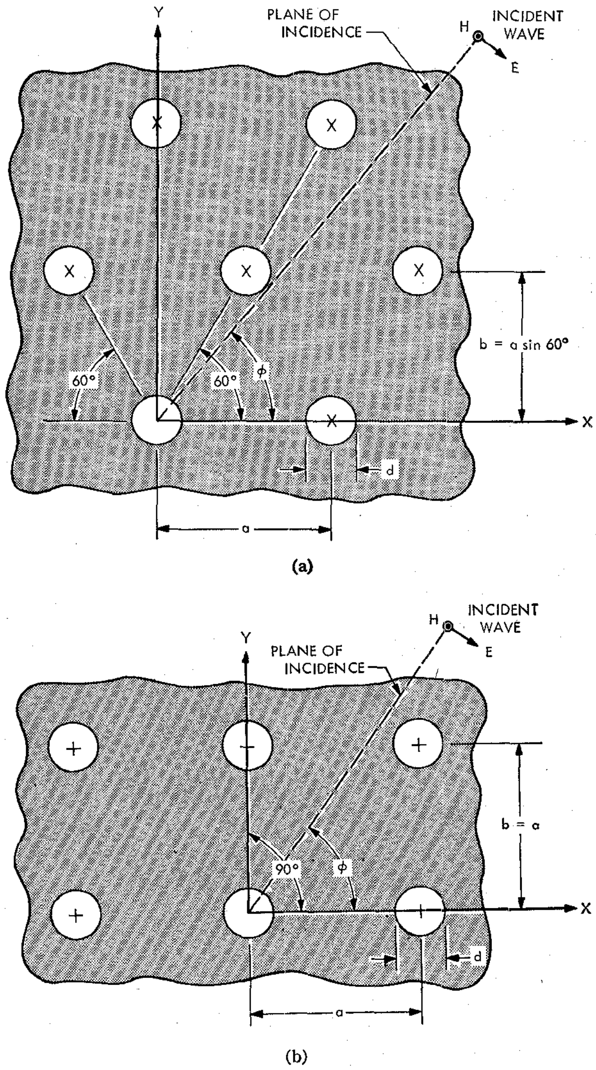

Fig. 1. Geometry for two-dimensional array of holes in metallic flat plate. (a) 60° (staggered) hole pattern configuration. (b) 90° (square) hole pattern configuration.

and $d < a, b \ll \lambda_0$. It should be pointed out that the quantity in parentheses in (1) is equivalent to the first term shown in Culshaw [9, eq. (25)]. The summation term in [9, eq. (25)] is a small correction term that can generally be neglected when $a, b, d \ll \lambda_0$.

Generalizing (1) to the case of an obliquely incident plane wave with the E field polarized normal to the plane of incidence and also accounting for the effect of plate thickness, the approximate expression for transmission loss[1] is

$$T_{\text{dB}} = 20 \log_{10}\left(\frac{3ab\lambda_0}{2\pi d^3 \cos\theta_i}\right) + \frac{32t}{d} \tag{2}$$

where

- θ_i angle of incidence
- t plate thickness

and $d < a, b \ll \lambda_0$.

The last term of (2) is a plate thickness correction term that was derived by analyzing the basic cell of the circular hole array as a π network and by treating the small hole as a circular waveguide beyond cutoff. The treatment follows that given by Marcuvitz [11] except that, for the derivation of (2), the expression for normalized susceptance applicable to the circular hole array was used.

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[1] The term transmission loss as used here is equivalent to the term attenuation defined by Beatty [10]. It is the insertion loss of the mesh when placed in a nonreflecting system.